# Measured $TE_{01}$ Attenuation in Helix Waveguide with Controlled Straightness Deviations

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*A helix circular waveguide 380 feet long has been deformed into three different curves. The added mode conversion loss for a $TE_{01}$ signal mode has been measured and calculated theoretically, with good agreement. For a curve with 30-inch deflection over a distance of 100 feet, peak-to-valley (200-ft period), the measured added loss at 55.5 kmc was 0.23 db/mile, the calculated 0.20 db/mile.*

## I. INTRODUCTION

The added mode conversion loss in a circular waveguide due to random straightness deviations of the guide axis has been calculated by Rowe and Warters.[1] These calculations show that if the mechanical spectrum of the axis wiggles has high density near the beat wavelengths of the coupling modes, then the transmission loss for a $TE_{01}$ signal mode can be excessive for axis deviations of fractions of a mil. However, as emphasized in this paper and in other calculations,[2,3] the requirement on the straightness depends strongly on the mechanical spectrum of the deviations. If the significant part of the mechanical spectrum is far from the beat wavelength of the coupling modes, then extremely large deviations (several feet) do not seriously affect the $TE_{01}$ transmission. Here we offer experimental evidence of these facts for a 2-inch diameter helix waveguide, and associated theoretical calculations which agree favorably with the experimental results.

In Section II we describe the experiment and discuss briefly the measured results of the experiment. Section III contains some theoretical calculations and, finally, in Section IV we discuss the comparison of experimental and theoretical results and some conclusions of the experiment.

## II. DESCRIPTION OF EXPERIMENT AND EXPERIMENTAL RESULTS

A steel-jacketed helix waveguide 380 feet long which was constructed at the Holmdel, N.J., Bell Laboratories[4] was used in this experiment. The guide was bent in the vertical plane to conform to three different curves. A photograph of the guide in one case appears in Fig. 1. The first two curves were roughly sinusoidal and so had a very limited spectral content. The first curve had a period of 40 feet, and the peak-to-peak deflection was 2.4 inches. The second curve had a period of 200 feet and a peak-to-peak deflection of 30 inches. The third curve was a simple model of a deviation which might be caused by random laying errors; basically, the guide was supported every 10 feet by a support which was either 1.2 inches higher or lower than the previous support (mechanical nonuniformities such as the brass couplings prevented rigid adherence to this format at four supports). The actual displacements are given in Table I.

The electrical measurements were performed with the above-ground waveguide test set at Holmdel, using the shuttle-pulse technique.[5] The measured $TE_{01}$ loss over the 50–60 kmc band is shown on an expanded scale in Fig. 2. The measured added losses were 0.65 db/mile

Fig. 1 — 30-inch peak, 200-foot period bend.

TABLE I — ACTUAL DISPLACEMENTS IN EXPERIMENTAL 380-FOOT HELIX WAVEGUIDE

| $y$ (inches) | $x$ (feet) | $y$ (inches) | $x$ (feet) | $y$ (inches) | $x$ (feet) |
|---|---|---|---|---|---|
| 0.0 | 0 | 1.2 | 130 | 2.2 | 260 |
| 0.6 | 10 | 2.4 | 140 | 2.4 | 270 |
| 1.2 | 20 | 1.2 | 150 | 3.6 | 280 |
| 1.2 | 30 | 0.0 | 160 | 4.8 | 290 |
| 0.0 | 40 | 0.0 | 170 | 3.6 | 300 |
| 1.2 | 50 | 1.2 | 180 | 2.4 | 310 |
| 2.4 | 60 | 2.4 | 190 | 1.2 | 320 |
| 3.6 | 70 | 1.2 | 200 | 0.0 | 330 |
| 2.4 | 80 | 0.0 | 210 | 0.0 | 340 |
| 1.2 | 90 | 1.2 | 220 | 1.2 | 350 |
| 1.2 | 100 | 2.4 | 230 | 0.6 | 360 |
| 0.0 | 110 | 1.8 | 240 | 0.0 | 370 |
| 0.0 | 120 | 2.4 | 250 | 1.2 | 380 |

0.23 db/mile, and 0.30 db/mile over the 2.55 db/mile measured at 55.5 kmc with the waveguide straight. The smallest added loss was associated with the bend with the largest amplitude and longest period, thus emphasizing that slow bends cause little added loss even for quite large deviations of the guide axis.

## III. THEORY

Calculation of the added mode conversion loss requires three steps: (1) calculation of the curvature of the guide; (2) calculation of the wall

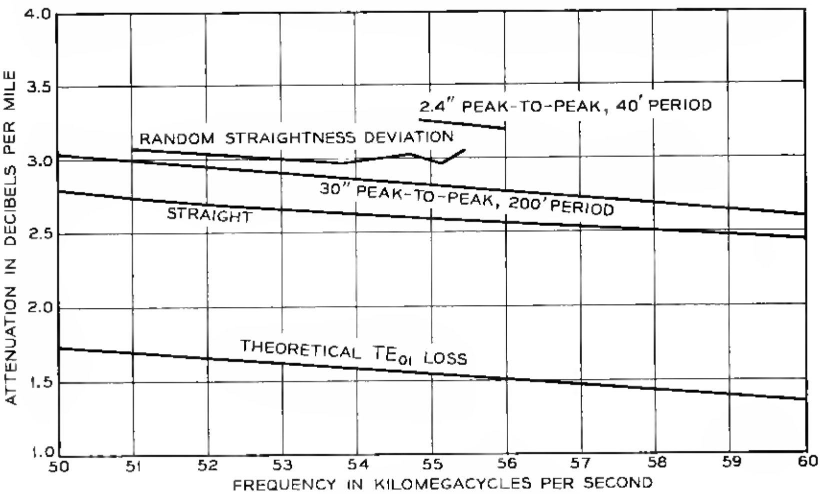


Fig. 2 — Measured $TE_{01}$ attenuation.

impedance in order to determine the propagation constants and coupling coefficients; (3) application of the Rowe-Warters perturbation calculation.

For the curvature calculation we assume the following mechanical model: a uniform homogeneous circular tube of outer radius $b$ and inner radius $a$ with a modulus of elasticity $E$ and a distributed weight $\omega_0$ pounds/foot. The moment of inertia about the axis is $I = (\pi/4)(b^4 - a^4)$. The tube is supported by $(N + 1)$ point supports a distance $l$ apart and the ends are free, so that $y''(0) = y''(Nl) = 0$, where $y(x)$ is the vertical displacement from a preselected reference. Now the simple bending equation for beams is

$$1/R = M/EI \tag{1}$$

where $R$ is the radius of curvature, $M$ is the bending moment and $1/R = y''/[1 + (y')^2]^{\frac{3}{2}}$. For the displacements of interest here $|y'| \ll 1$ and $(1/R) \approx y''$. Now we assume further that the external forces at the supports are concentrated at discrete points. Then, if we differentiate (1) twice, we have

$$EIy^{IV} \approx -\omega_0 + \sum_{i=1}^{N+1} F_i\delta(x - x_i) \tag{2}$$

where $F_i$ is the force at the $i$th support, $\delta(x - x_i)$ is the Dirac delta function and $x_i$ is the location of the $i$th support.

The $F_i$ are difficult to measure and (2) cannot be solved in terms of a simple analytic function. However, if we consider the equation between supports the analytic form of the solution is immediate, and we need to use our end and continuity conditions to evaluate the necessary constants.

Let $z_i = x - x_i$ and $0 \leqq z_i \leqq l$ for all $i$; then for the $i$th section (2) becomes

$$y_i^{IV}(z_i) = -\omega_0/EI \tag{3}$$

which has the solution

$$y_i(z_i) = -\frac{\omega_0}{24EI} z_i^4 + A_i z_i^3 + B_i z_i^2 + C_i z_i + D_i\,. \tag{4}$$

The conditions necessary to obtain the constants are: $y_1''(0) = 0$, $y_N''(l) = 0$ and $y_i$, $y_i'$, $y_i''$ continuous at the supports. With $N + 1$ supports we have to evaluate $4N$ constants. The continuity conditions are $3(N - 1)$ in number. These, plus the $N + 1$ known displacements at the supports and the two end conditions, yield a total of $4N$ conditions. The actual calculation is long, but straightforward.

H. -G. Unger[6,7] has solved the helix waveguide problem of calculating propagation constants and curvature coupling coefficients assuming a model with the following boundary conditions at the helix:

$$\begin{aligned} E_\theta &= 0 \\ E_z &= -ZH_\theta \, . \end{aligned} \tag{5}$$

Unger's analysis gives a very general solution if the wall impedance $Z$ due to a complex jacket outside the helix can be calculated. Strictly speaking, the wall impedance will depend on the mode through the propagation constant, so that an exact solution is not possible, but in the oversize waveguide of interest here the wall impedance may be calculated by assuming the propagation constant of each mode equal to the propagation constant of free space. For the modes of interest which couple to the $TE_{01}$ mode due to curvature, the angular wave number is small compared to the longitudinal and radial wave numbers, and neglecting angular variation allows a great simplification by permitting consideration of the jacket structure in rectangular coordinates with no variations of the field in the plane of the helix wires.

For the present helix, the jacket structure consists of a thin layer of clear glass-fiber roving followed by a thicker layer of material that is an aqueous formulation of graphite bonded to continuous filaments of glass roving. A universal wrap gives a checkerboard array with the fibers oriented at about $\pm 45°$ with respect to the $z$ axis of the guide. These layers are enclosed in a steel jacket and are impregnated with epoxy resin. We assume an equivalent transmission line circuit for this jacket, as shown in Fig. 3. $Z_c$ is the shunt capacitance due to the helix wires, region 1 is the lossless layer, region 2 the lossy layer, and the line is terminated by the short circuit of the steel jacket. We calculate the impedance and propagation constant of region 2 from a model used by S. E. Miller,[8] which assumes an infinite lossless region of fiber glass in which are imbedded parallel resistive fibers of zero cross section, spaced uniformly a distance $t$ from each other. The conductance of the fibers is deduced from the dc resistance of the carbon-coated glass roving. We multiply the measured conductance by $\cos^2 45°$ to include the effect of the $\pm 45°$ wrap. The formulas for the impedances and propagation constants are:

($i$) helix-wire capacitance:[6]

$$Z_c = \frac{1}{j\omega\epsilon \, d\left(\dfrac{d}{D - d} - \dfrac{\ln 4}{\pi}\right)} \tag{6}$$

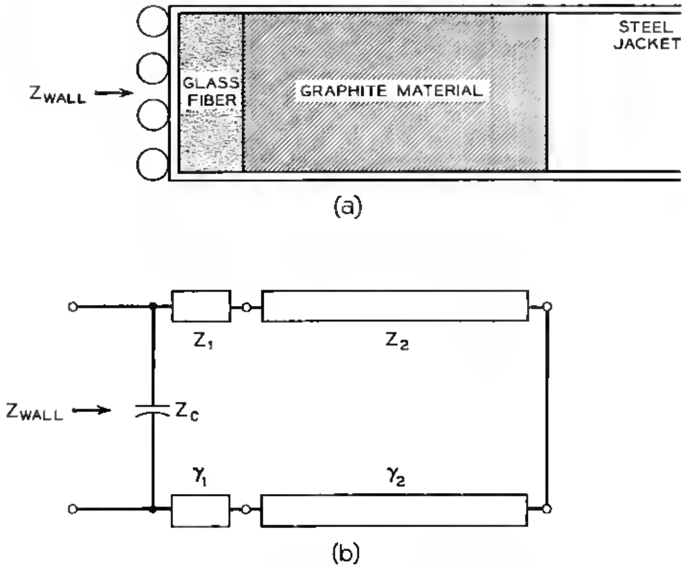


Fig. 3 — (a) Physical structure of helix waveguide used in experiment; (b) equivalent transmission line circuit.

where we have

$D$ = 0.0055 inch (diameter of helix wires including insulation)

$d$ = 0.0045 inch (diameter of helix wires)

$\epsilon = 2.77\epsilon_0$ (permittivity of Formvar insulation)

(*ii*) lossless layer: $\epsilon = 4\epsilon_0$, $l_1 = 0.005$ inch

$$Z_1 = \sqrt{\frac{\mu_0}{4\epsilon_0}} = \frac{1}{2} Z_0$$

$$\gamma_1 = \omega\sqrt{4\mu_0\epsilon_0}$$

(*iii*) lossy layer:[8] $\epsilon = 4\epsilon_0$; $l_2 = 0.033$ inch

$$Z_2 = \frac{1}{\dfrac{\cos^2 45^\circ}{2R} + \left\{\dfrac{4\epsilon_0}{\mu_0} + \dfrac{1}{4}\left[\dfrac{\cos^2 45^\circ}{R}\right]^2 - j\,\dfrac{\cos^2 45^\circ \sqrt{\dfrac{4\epsilon_0}{\mu_0}}}{R \tan \omega\sqrt{4\mu_0\epsilon_0}\, t}\right\}^{\frac{1}{2}}}$$

$$\gamma_2 = (1/t) \ln \left\{ \cos \omega\sqrt{4\mu_0\epsilon_0}\, t \left[ 1 + \frac{\cos^2 45^\circ}{R\left(\dfrac{1}{Z_2} - \dfrac{\cos^2 45^\circ}{R}\right)} \right.\right.$$

$$+\frac{j\sqrt{\frac{4\epsilon_0}{\mu_0}}\tan\omega\sqrt{4\mu_0\epsilon_0}\,t}{\left(\frac{1}{Z_2}-\frac{\cos^2 45°}{R}\right)}\Bigg]\Bigg\}$$

where $t = 0.0081$ inch (average spacing of resistive fibers) and $R = 234$ ohms/square (from dc resistance measurement). These parameters correspond to a wall impedance at 55.5 kmc of

$$\frac{Z_{\text{wall}}}{Z_0} = 0.41\ \angle -32°$$

where $Z_0 = \sqrt{\mu_0/\epsilon_0}$.

From the wall impedance the coupling coefficients and propagation constants of the spurious modes can be determined.[7] Then the $TE_{01}$ loss due to mode conversion can be computed approximately using the method of Picard developed by Rowe and Warters.[1] Summing over all spurious modes $k$, we have:

$$A(\text{nepers}) = \sum_{k=1}^{\infty} \text{Re}\left[\int_0^L e^{\Delta\Gamma_k u}\,du \int_0^{L-u} c_k(x)c_k(x+u)\,dx\right] \quad (8)$$

where

$$\Delta\Gamma_k = \Delta\alpha_k + j\Delta\beta_k = \Gamma_{TE_{01}} - \Gamma_k$$

and

$$\begin{aligned} c_k(x) &= (c_k R)y''(x) \\ &= (c_k' + jc_k'')Ry''(x). \end{aligned}$$

The sum is over the infinite number of coupling modes; however convergence of the coupling coefficients is rapid, and only three modes contribute significantly, $TE_{11}$, $TE_{12}$, and $TM_{11}$. Since each $\Delta\alpha$ is large and negative and the slowness of the bends makes the inner integral (8) a slowly varying function of $u$, we have,

$$\int_0^L e^{\Delta\Gamma_k u}\,du \int_0^{L-u} c_k(x)c_k(x+u)\,dx \approx \int_0^L e^{\Delta\Gamma_k u}\,du \int_0^L c_k^2(x)\,dx. \quad (9)$$

With this approximation and the fact that $e^{\Delta\alpha_k L} \ll 1$, (8) becomes

$$A \approx \int_0^L [y''(x)]^2 dx \sum_{k=1}^{\infty} \frac{P_k(-\Delta\alpha_k) - Q_k\Delta\beta_k}{(\Delta\alpha_k)^2 + (\Delta\beta_k)^2} \quad (10)$$

where

$$P_k = (c_k'^2 - c_k''^2)R^2$$
$$Q_k = +2c_k'c_k''R^2.$$

To evaluate

$$\int_0^L [y''(x)]^2 dx$$

we need the second derivative of (4):

$$\int_0^L [y''(x)]^2 dx = N\left[\frac{\omega_0}{2EI}\right]^2 \frac{l^5}{5} - \frac{\omega_0 l^4}{4EI}\sum_{i=1}^N A_i + \frac{l^3}{3}\sum_{i=1}^N \left(A_i^2 - \frac{\omega_0}{EI}B_i\right) + l^2 \sum_{i=1}^N A_iB_i + l\sum_{i=1}^N B_i^2. \tag{11}$$

The $A_i$ and $B_i$ are obtained from the mechanical boundary conditions. After eliminating the $C_i$ and $D_i$ from (4), we have the following $2N - 1$ equations in $A_i$ and $B_i$:

$$\begin{bmatrix} A_1 \\ A_2 \\ \vdots \\ A_N \end{bmatrix} = \frac{1}{l}\begin{bmatrix} B_2 - 0 + \frac{\omega_0}{2EI}l^2 \\ B_3 - B_2 + \frac{\omega_0}{2EI}l^2 \\ \vdots \\ 0 - B_N + \frac{\omega_0}{2EI}l^2 \end{bmatrix} \tag{12}$$

$$\begin{bmatrix} 4100 \cdots\cdots \\ 1410 \cdots\cdots \\ 0141 \cdots\cdots \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ \cdots\cdots\cdots \\ \cdots\cdots 1410 \\ \cdots\cdots 0141 \\ \cdots\cdots 014 \end{bmatrix} \begin{bmatrix} B_2 \\ B_3 \\ \cdot\cdot \\ \cdot\cdot \\ \cdot\cdot \\ \cdot\cdot \\ B_{N-2} \\ B_{N-1} \\ B_N \end{bmatrix} = \frac{6}{l^2}\begin{bmatrix} y_3 - 2y_2 + y_1 - \frac{\omega_0}{12EI}l^4 \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ \cdots\cdots\cdots\cdots \\ y_{N+1} - 2y_N + y_{N-1} - \frac{\omega_0}{12EI}l^4 \end{bmatrix} \tag{13}$$

TABLE II — WALL IMPEDANCES

| | $\Delta\alpha$(nepers/ft) | $\Delta\beta$(radians/ft) | $P$ | $Q$ |
|---|---|---|---|---|
| $TE_{11}$ | −0.072 | −1.87 | 45.5 | 2.3 |
| $TM_{11}$ | −0.560 | +2.11 | 31.4 | 22.3 |
| $TE_{12}$ | −0.133 | +2.94 | 65.6 | −24.6 |

where $y_i$ is the measured displacement of the guide at the $i$th support. The inverse of the square matrix in (11) may be found by solution of difference equations and symmetry properties. The inverse matrix has the following elements:

$$a_{ik} = c_1 \alpha_1^{k-i+1} \frac{1 - \alpha_1^{2i}}{1 - \alpha_1^2} + c_2 \alpha_2^{k-i+1} \frac{1 - \alpha_2^{2i}}{1 - \alpha_2^2} \qquad \text{for} \quad i \leqq k \leqq N - i + 1, \quad i < N/2 \tag{14}$$

where

$$\alpha_1 = -2 + \sqrt{3}$$

$$\alpha_2 = -2 - \sqrt{3}$$

$$c_1 = \frac{\alpha_1^N}{\alpha_2^N - \alpha_1^N}$$

$$c_2 = \frac{\alpha_2^N}{\alpha_1^N - \alpha_2^N}$$

and the remaining elements are obtained from the symmetry about the main diagonal and the off diagonal.

The wall impedance 0.41 $\angle -32°$ corresponds to the values given in Table II. The computed loss and the experimentally measured loss are listed in Table III.

TABLE III — COMPUTED AND EXPERIMENTALLY MEASURED LOSSES

| Case | Period | Max. Deflection | Added Exp. Loss | Added Cal. Loss |
|---|---|---|---|---|
| 1 | 40 feet | 2.4 inches | 0.65 db/mile | 0.69 db/mile |
| 2 | 200 feet | 30.0 inches | 0.23 db/mile | 0.20 db/mile |
| 3 | — | 2.4 inches | 0.30 db/mile | 0.27 db/mile |

## IV. RESULTS AND CONCLUSIONS

The experimental results agree favorably with the computed results, considering the various approximations in the theoretical model. The important result is that mechanical deviations of long wavelengths, such as we might expect from random laying errors, contribute very little to the transmission loss. Thus the tolerance on guide axis wiggles changes several orders of magnitude when the mechanical wavelength changes from two feet to two hundred feet.

## V. ACKNOWLEDGMENTS

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